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Elements for the Agent-Based Modeling of Slavery Systems

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KEYWORD

Agent-basec; Modeling; Slavery-based economic systems; Agent societies

ABSTRACT

This paper introduces formal concepts for the agent-based modeling of slavery systems. The concepts of master-slave economic relationship, slavery-based economic system, slavery-supporting legal system, and slavery-based material agent society are formally defined. A first case study recasts, for material agent societies, North & Thomas' economic model determining the objective conditions under which it is rational for a society to choose a slavery-based economic system over a free labor-based economic system. A second case study makes use of elements of F. H. Cardoso's study of slavery in the south of Brazil to illustrate the application of the formal concepts introduced in the paper.

1. Introduction

In this paper, we introduce a conceptual framework supporting the agent-based modeling of *general slavery systems* (see, e.g., (Archer, 1988) for a general account of the history and varieties of slavery). *Material agent societies* and their elementary economic systems (Costa, 2018) are taken as the formal societal context in which general slavery systems are modeled.

The *social* and *economic master-slave relationships* that characterize *slavery systems*, the *economic systems* that arise from them, and the *legal systems* that support them, are informally explained and formally defined. A formal general definition is given of *slavery-based material agent societies*.

So called *chattel slavery* is assumed as the basic form of slavery, and is formally characterized. A case study recasts North & Thomas' economic model determining the objectives conditions under which it is rational for a society to choose a *slavery-based economic system* over a *free labor-based economic system*.

The aim of the present paper, in line with the work that has been done concerning *agent societies* in general (Costa, 2017c; Costa, 2019), and *material agent societies* in particular (Costa, 2017b; Costa,

2018), is to provide an *agent-based semantical model* for formal social and political theories of *slavery-based societies*.

However, we investigate no single theory of slavery-based societies, regarding such applicability. In particular, we do not go beyond the general level of detail of the concepts mentioned above.

2. Agent Societies

A detailed formal account of the architecture of material agent societies that are organized on the basis of slavery is out of the scope of the present paper. Here, we can only leave implicit the amount of details that would have to be provided in order to properly instantiate, as a *slavery-based material agent society*.

We take the following as the general structure of an *agent society* (Costa, 2017c):

- *AgSoc* = (*Pop*, *Org*, *MEnv*, *SEnv*, *IMP*, *ACC*)

where:

- *Pop* is the *population* of *AgSoc*;
- *Org* is the *organizational structure* of *AgSoc*;
- *MEnv* is the *material environment* of *AgSoc*;
- *SEnv* is the *symbolic environment* of *AgSoc*, where the society's system of *legal norms* is embedded;
- *IMP* is the collection of *implementation relations* between *Pop* and *Org*;
- *ACC* is the collection of *access relations* of *Pop* and *Org* to the environments *MEnv* and *SEnv*.

Moreover, considering the case of slavery-based material agent societies that *do not* produce slaves by capturing them in another society, but that, besides producing them by parental reproduction, also *import* them from *slave capturing societies*, the full account of the details of that slavery-based society would require placing it in an *inter-societal context* (see (Costa, 2017a)), which is also out of the scope of the present paper.

3. Elementary Economic Exchanges in Material Agent Societies

As in (Costa, 2017b) and (Costa, 2018), we say that an agent is a *material agent* whenever that agent has a *material body*, that is, a body that requires *energy* for its operation. And we call *material agent society* any agent society whose agents are all material agents.

We call *energy producer* any material agent that is capable of producing *energy objects*, with which energy is distributed for consumption in the society. All the other material agents of that society are said to be *energy consumers*.

Formally, we denote:

- *EnergProd*: the set of material agents that are *energy producers*;
- *EnergCons*: the set of material agents that are *energy consumers*.

Two types of *objects* are assumed to be exchangeable in an *elementary economic exchange* between two material agents:

- *EnergObj*, the type of the so-called *energy objects*, that is, objects that are carriers of the energy needed by the material agents for their functioning;
- *Chip*, the type of the so-called *chips*, that is, valuable objects that the material agents may be interested to acquire, possibly by exchanging some of the *energy objects* they have in their possession;

so that, in general, an *elementary economic exchange* is constituted by an exchange between two material agents where:

- either one or more *energy objects* are exchanged for one or more *chips*;
- or one or more *chips* of a given subtype are exchanged for one or more *chips* of another subtype. We will be mainly interested in elementary economic exchanges of the first kind.

4. Slavery-Based Economic Systems

4.1. Masters and Slaves in Chattel Slavery

Chattel slavery is slavery where *slaves* are considered personal belongings of their *masters*, which can dispose of them as they wish. Other forms of slavery also exist (see, e.g., (Archer, 1988)).

In this paper, we take into account only *chattel slavery*. However, we make an informal use of the term *property*, that is, we use property to mean both *informal*, non-legally supported ownership of objects, as well as *formal*, legally supported ownership of objects, *slaves* being a particular type of ownable objects (Blackburn, 1988, p.276):

> «*the slave status and condition has been a purely social construction—that of a social isolate, an outsider, a person without kin, a person subject to the complete and arbitrary authority of the master, a person who could be whipped or tortured or sexually abused, a piece of property, and, by virtue of the foregoing, an instrument. The very enumeration of such qualities must remind us that slavery was not a suprahistorical essence but had to be produced and perpetuated, enlisting the support of the free population and adapting the slave to the particular use required.*»

Given a material agent society *MatAgSoc*, we formally define[1]

- *MatObj*: the set of *material objects* of *MatAgSoc*;
- $MatAg \in \wp(MatObj)$: the set of *material agents* of *MatAgSoc*;
- $Master \in \wp(MatAg)$: the set of *masters* of *MatAgSoc*;
- $Slave \in \wp(MatAg)$: the set of *slaves* of *MatAgSoc*. For simplicity, we take that:
- $Master \cap Slave = \varnothing$: no master is a slave, and vice-versa;
- $Master \in \wp(EnergCons)$: all masters are *energy consumers*;
- $Slave \in \wp(EnergProd)$: all slaves are *energy producers*.

[1] $\wp(X)$ is the powerset of set X.

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In the following sections, we introduce formal accounts of diverse aspects of the *property relationship* between masters and slaves.

4.2. Master-Slave Property Relationship

We call *master-slave property relationship* the system of actions, norms and commands that empower the set of masters of a *slave-based material agent society*, so that they can maintain slaves in their slavery condition.

Formally, we characterize the *property relationship* between masters and slaves in the following way:

- $owns \subseteq Master \times Slave$, the *property relation* between masters and slaves, so that $owns(master_i, slave_j)$ means that master $master_i$ owns $slave_j$.

The *property relationship* allows $master_i$ and $master_k$ to do any of the following actions on $slave_j$, whenever it happens that $owns(master_i, slave_j)$:

- $sell\ (master_i, slave_j, master_k)$
- $lend\ (master_i, slave_j, master_k)$
- $rent(master_i, slave_j, master_k)$
- $kill\ (master_i, slave_j)$
- $free(master_i, slave_j)$
- $command\ (master_i, slave_j, cmd)$
- $punish(master_i, slave_j, cmd)$

meaning that:

- $master_i$ can *sell*, *lend* and *rent* $slave_j$ to any other master $master_k$, besides *killing* or *freeing* it, and *commanding* it to perform any command *cmd*, as well as *punishing* it for the way it performed such command.

In accordance with the possibility of $master_i$ performing $sell\ (master_i, slave_j, master_k)$, we take that $master_k$ can *buy* slave $slave_j$ from master $master_i$. That is, we also have, as possible action:

- $buy\ (master_k, slave_j, master_i)$

Additionally, the following (formal or informal) obligation is taken to be valid for slaves:

- $mustexec(slave_j, cmd, master_i)$ meaning that:
- $slave_j$ is supposed to peremptorily execute any command *cmd* issued by $master_i$.

For simplicity, we omit here any reference to the *conditions* under which those *actions* and *commands* may be effective, such as the explicit *connection* between *commands* and possible *punishments*. But, see Section 5 for some of the *legal* types of such conditions.

Other legal forms of *acquisition* of slaves (such as by having them *born* from parents that are already slaves, or by *capturing* them in certain specified conditions) are also considered in Section 5.

4.3. Master-Slave Economic Exchanges

Let time be given by the linearly ordered set $T = \{0, 1, 2,...\}$, ranged over by variable t. For the purpose of the present paper, we call *elementary economic exchange* any exchange of the form (cf. (Costa, 2018)):

$$e2exch^t = \langle mag / obj \rangle^t \; {}_{c'}\!\rightleftarrows^{c} \; \langle mag' / obj' \rangle^t$$

meaning that material agents *mag* and *mag'* exchange objects *obj* and *obj'*, a pair of such objects at each time *t*, under the assumption that *mag* provides operational condition *c* for *mag'* to produce and delivery *obj'*, and *mag'* provides operational condition *c'* for *mag* to produce and delivery *obj*.

We call *master-slave elementary economic exchange* any elementary economic exchange of the form:

$$mse2exch^t = \langle master / \perp \rangle^t \; {}_{\perp}\!\rightleftarrows^{c} \; \langle slave / obj \rangle^t$$

meaning that, at each *t*, *slave* sends an object *obj* to his master *master,* without receiving no object in exchange ($\perp$ is the *null* object), while the master *master* is required to provide condition *c* for *slave* to produce and delivery *obj*, and the slave *slave* is not required to provide any condition ($\perp$) for the master to produce and delivery nothing ($\perp$).

Notice that in any *master-slave elementary economic exchange* like *mse2exch*^{t}, masters accumulate a set of received objects, up to time *t*, in the form:

$$accumobj[master / mse2proc^t] = \{obj^0, obj^1, \ldots, obj^t\}$$

while slaves accumulate nothing, because we take that a set of *null* objects is an empty set. That is:

$$accumobj[slave / mse2proc^t] = \{\perp^0, \perp^1, \ldots, \perp^t\} = \emptyset$$

In general, *masters* are allowed to have a *group of slaves* with more than one slave in it. In such a case, the *master-slave group elementary economic exchange* that the *master* and the *group of slaves* perform has the form:

$$msge2exch^t = \langle master / \perp \rangle^t \; {}_{\perp}\!\rightleftarrows^{c} \; \langle Slave / Obj \rangle^t$$

where:

- *Slave* is the *group of slaves* that belong to *master*;
- *Obj* is the *set of objects* that the set of slaves *Slave* produce and deliver, at each time, to *master.*

4.4. Slavery-Based Elementary Economic Processes

In general, *individual elementary economic processes* have the form (see (Costa, 2018)):

$$ie2proc^t = \langle mag_1 / \perp, obj_{1,2} \rangle^t \; {}_{c_{2,1}}\!\rightleftarrows^{c_{1,2}} \; \langle mag_2 / obj_{2,1}, obj_{2,3} \rangle^t \; {}_{c_{3,2}}\!\rightleftarrows^{c_{2,3}} \; \ldots$$
$$\ldots \; {}_{c_{n,n-1}}\!\rightleftarrows^{c_{n-1,n}} \; \langle mag_n / obj_{n,n-1}, \perp \rangle^t$$

where:

- mag_i is the i-th material agent participating in $ie2proc^t$;
- each $\langle mag_i / obj_{i,i-1}, obj_{i,i+1}\rangle^t \; c_{i+1,i} \rightleftarrows^{c_{i,i+1}} \langle mag_{i+1} / obj_{i+1,i}, obj_{i+1,i+2}\rangle^t$ is an *elementary economic exchange;*
- $obj_{i,k}$ is the object that mag_i produces and deliveries to its k-th partner, for $k \in \{i - 1, i + 1\}$;
- mag_1 has no left partner, so $obj_{1,0} = \perp$;
- mag_n has no right partner, so $obj_{n,n+1} = \perp$.

In the general case of *slavery-based elementary economic processes*, we have the form:

$$mse2proc^t = \langle master_1 / \perp, obj_{1,2}\rangle^t \; c_{2,1} \rightleftarrows^{c_{1,2}} \langle master_2 / obj_{2,1}, obj_{2,3}\rangle^t \; c_{3,2} \rightleftarrows^{c_{2,3}} \ldots \\ \ldots \; c_{n,n-1} \rightleftarrows^{c_{n-1,n}} \langle master_n / obj_{n,n-1}, \perp\rangle^t$$

where one can notice that only *masters* participate in the society's economic processes, *slaves* being restricted to private economic exchanges with their *masters*, as sketched in Figure 1. Notice that a *master* participates in *two* elementary economic exchanges (one local, with his *slave*, the other global, with other *masters*), while a *slave* participates only in *one* elementary economic exchange, the local exchange with its *master*.

Notice also that this model of slavery-based elementary economic processes naturally extends to the cases where the masters may have *groups of slaves*, instead of just individual slaves.

4.5. Slavery-Based Elementary Economic Systems

Let the term *elementary economic material agent* denote a material agent of the population of a material agent society that can participate in an elementary economic exchange. We define the general notion of *elementary economic system* of a material agent society as follows (cf. (Costa, 2018)):

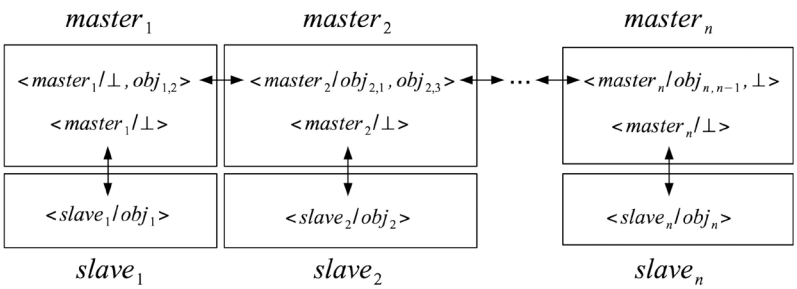

Figure 1: Sketch of a slavery-based elementary economic process.

Definition 4.1 *The* elementary economic system *E2Sys of a material agent society MAgSoc is a time-indexed structure:*

$$E2Sys^{\,t}_{MAgSoc} = (E2MAg^t, Objs^t, E2Beh^t, E2Exch^t, E2Proc^t)$$

where, for each time t:

- *E2MAg^t is the set of* elementary economic material agents*, which participate in the elementary economic processes of MAgSoc, at the time t;*
- *Objs^t is the set of* objects *that the elementary economic agents can exchange during the performance of their elementary economic exchanges, at that time;*
- *E2Beh^t is the set of* elementary economic behaviors *that the elementary economic material agents can perform during the performance of the elementary economic exchanges, at that time;*
- *E2Exch^t is the set of* elementary economic exchanges *that the elementary economic material agents can perform during the performance of the elementary economic processes, at that time;*
- *E2Proc^t is the set of* elementary economic processes *that the elementary economic material agents can perform in MAgSoc, at that time.*

In the case of a *slavery-based elementary economic system*, we have that, for any time *t*:

- *E2MAg^t = Master^t ∪ Slave^t*, meaning that the set of elementary economic material agents of *E2Sys^t* is partitioned into *masters* and *slaves*;
- *Obj^t = MasterSlaveObj^t ∪ MasterMasterObj^t*, meaning that the objects exchanged between elementary economic agents are either of the *master-slave* type or of the *master-master* type, with (see (Costa, 2018)):
  - *MasterSlaveObj^t ∈ ℘(EnergObj)*, that is, objects exchanged between masters and slaves (in fact, just from slaves to masters) are *energy objects*, resulting from the labor of the slaves;
  - *MasterMasterObj^t ∈ ℘(EnergObj) ∪ ℘(Chip)*, that is, objects exchanged between masters are either *energy objects* or *chips*, with chips exchanged in return for energy objects;
- *E2Beh = MasterE2Beh^t ∪ SlaveE2Beh^t*, that is, both masters' and slaves' elementary economic behaviors may participate in the elementary economic exchanges of $E2Sys^t_{\text{MAgSoc}}$;
- *E2Exch = MasterSlaveE2Exch^t ∪ MasterMasterE2rExch^t*, that is, the elementary economic exchanges are either of the *master-slave* type or of the *master-master* type;
- *E2Proc = MasterSlaveE2Proc^t ∪ MasterMasterE2Proc^t*, that is, the elementary economic processes are either of the *master-slave* type or of the *master-master* type.

Notice that no particular requirement is established concerning the types of *conditions* that may be imposed on the elementary economic exchanges.

5. Slavery-Supporting Legal Systems

5.1. Legal Systems of Agent Societies

We define the general notion of *legal system* situated in a general agent society *AgSoc* as follows (cf. (Costa, 2015)):

Definition 5.1 *A* legal system *situated in AgSoc is a* time-indexed structure:

$$LegalSys^t_{AgSoc} = (LOrd^t, LOrg^t, RLFact^t, LegalOps)$$

where:

- $LOrd^t$ *is the* legal order *at time t;*
- $LOrg^t$ *is the* system of legal organs *at time t;*
- $RLFact^t$ *is the* record of legal facts *at time t;*
- *LegalOps is the set of* legal operations*, like:*
  - *createlnrm, the* creation *of legal norms;*
  - *deroglnrm, the* derogation *of legal norms;*
  - *createlauth, the* creation *of authorizations to perform* legal operations*;*
  - *cancellauth, the* cancellation *of such authorizations;*
  - *recordlfct, the* recording *of a legal fact in RLFact;*
  - *deletelfct, the* deletion *of one such record.*

with:

- *RLFact freely accessible to all the agents of the society;*
- $LOrd^t$ *and* $LOrg^t$ *such that* $LOrd^0 \neq \varnothing \neq LOrg^0$.

5.2. Legal Systems in Slavery-Based Material Agent Societies

In the context of *slavery-based material agent societies* endowed with legal systems, we are particularly interested in the types of *legal norms* that support the masters in their maintenance of the *slave-based economic relationship*.

Cleraly, the most fundamental of such legal norms are:

- $owns(master_i, slave_j) \Rightarrow Auth(master_i, command\ (master_i, slave_j, cmd)$,
  which legally authorizes master $master_i$ to *command* that slave $slave_j$ does any command the master wishes;
- $owns(master_i, slave_j) \Rightarrow Auth(master_i, punish(master_i, slave_j, cmd)$,
  which legally authorizes master $master_i$ to *punish* slave $slave_j$ for not doing properly any command that the master has commanded it to do.

Typically, the following conditional legal authorization is also formally adopted by the legal systems of slave-based material agent societies, so that the initial condition for a material agent to become a slave, and for which is to be its initial master, is established:

- $owns(master_i, slave_j) \wedge mother\ (slave_j, mag_k) \Rightarrow Auth(master_i, owns(master_i, mag_k))$,
  which legally authorizes master $master_i$ to *own* the material agent mag_k as a slave, if the mother of mag_k is itself owned as a slave by master $master_i$.

Legal norms as the above ones, are sufficient for material agent societies whose only means to produce slaves is through their parental reproduction. Some material agent societies, however, adopt the legal procedure of allowing slaves to be produced by their capturing from other societies (either in the context of war between the two societies, or in the context of a commercial exploitation of the second society by the first one). That type of legal norm may be formally sketched as follows:

- $autorized\ (mag_i, slavecapture) \wedge captured\ (mag_i, mag_j) \Rightarrow Auth(mag_i, owns(mag_i, mag_{j)})$,

which states that if material agent mag_i is legally authorized to capture slaves, and it happened that mag_i captured mag_j then mag_i is legally authorized to own mag_j as a slave, effectively making of mag_i a *master* and of mag_j a *slave*.

6. A First Case Study: North & Thomas' Economic Model of Rational Choice between Slavery and Free Work in Material Agent Societies

6.1. Presentation

Douglass C. North and Robert Paul Thomas develop in (North and Thomas, 1971) an institutional dynamical model for the rise and fall of manorial systems, that encompasses both *serfdom* and *slavery*. They contrast *serfdom* an *slavery* by telling the former system to be «*essentially a contractual arrangement where labor services were exchanged for the public good of protection and justice*» (p.778), with a «*contractual relationship which could be changed only by both parties*» (p.779), while in the latter system a slave «*has no legal control over decision-making with respect either to his labor or to his income stream*» (p.779).

Clearly, by basing the distinction on the notion of *contract*, North & Thomas' model presupposes the existence of some sort of (formal or informal) *legal system* that is effective in the society and capable of enforcing the compliance with valid contracts.

The core of their rational model for the choice between *slavery-based* and *free labor-based* economic system is the following:

> «*Slavery was always more profitable than free labor <..> when the following conditions existed: (1) a market economy, (2) profitable opportunities to produce those types of economic activities where the costs of supervision to reduce shirking were low, and (3) where the costs of enforcing property rights in human beings were low.*» (p.779)

Notice that:

1. Condition (1) encompasses the possibility of freely *selling* and *buying* slaves.
2. Let $costsuperv(master_i, slave_j)$ denote the *cost of the supervision*, for master $master_i$, of the operation of the slave $slave_j$. Analogously, denote the cost of the corresponding supervision, concerning the free laborer *freelaborer k*, by $costsuperv(master_i, freelaborer_k)$. Then, condition (2) means that it is rational for the society to choose a *slavery-based economic system* over a *free labor-based economic system* if and only if, for a typical master (*mstr)*, it holds that:

$$\sum_{j=1}^{j=n} costsuperv(mstr, slave_j) \leq \sum_{k=1}^{k=m} costsuperv(mstr, freelaborer_k)$$

where:

- n is the *average number of slaves* owned by *mstr,* in the alternative of the *slavery-based economic system*;
- m is the *average number of free laborers* hired by *mstr,* in the alternative of the *free labor-based economic system*.

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3. Let *costenforc*(*master*$_i$, *slave*$_j$) denote the *cost*, for master *master*$_i$, of *enforcing property rights* on the slave *slave*$_j$ (both regarding the slave itself and the other competing masters). Analogously, let *costenforc*(*master*$_i$, *freelaborer* $_k$) denote the cost, for master *master*$_i$, of the corresponding enforcement, concerning the labor of the free laborer *freelaborer*$_k$. Then, condition (3) means that it is rational for a society to adopt a *slavery-based economic system* over a *free labor-based economic system* if and only if, for a typical master (*mstr)*, it holds that:

$$\sum_{j=1}^{j=n} costenforc(mstr, slave_j) \leq \sum_{k=1}^{k=m} costenforc(mstr, freelaborer_k)$$

where:

- *n* is the *average number of slaves* owned by *mstr,* in the alternative of the *slavery-based economic system*;
- *m* is the *average number of free laborers* hired by *mstr,* in the alternative of the *free labor-based economic system*.

In summary, one can see, by this brief account of the elements of North & Thomas' model, that a rational choice is possible between a *slavery-based economic system* and a *free labor-based economic system*, in any given *material agent society*, at any time of its history.

6.2. Discussion

The basic elements of North and Thomas' (North and Thomas, 1971) model for the rational choice between *slavery-based* and *free labor-based* economic systems was briefly reviewed. One sees that the analysis of their model opens the possibility for a *mixed* type of economic systems of material agent societies, namely, that which combines *slave material agents* and *free laborer material agents*.

Two criteria arise for a rational choice between *slavery* and *free laboring*, in such mixed situations:

- first, a choice at the level of the *type of economic activity*: choose between *slavery* and *free laboring* according to the costs of *work supervision* and *property rights enforcement* peculiar to each activity;
- second, a choice at the level of the *particular situation of the master*: choose between *slavery* and *free laboring* according to the costs of *work supervision* and *property rights enforcement* for each particular master.

A situation where the full combination of all such possibilities are adopted would certainly introduce extra complexity in the legal system of the society, because legal provisions would have to be established for each such possibility, including different legal norms applying to the same master, in accordance with the particular type of economic relation he has with each of his workers.

A more sensible choice would be that the material agent society chooses, for each *type of economic activity*, either *slavery* or *free labor*. In such case, the legal norms of the legal system, concerning the way work is performed in the material agent society, could be specialized for the different types of economic activities.

Finally, notice that even if the legal system of the society adopts only free labor based economic activities, it may be rational for some particular economic activity, or for some particular master, to establish slavery-based forms of economic exchange with its workers, giving rise to *illegal* slavery forms of economic activities in the society.

7. A Second Case Study: The Economic Fall of Slavery in Colonial South Brazil

7.1. The Colonial Situation

The *colonial* condition of Brazil, under Portuguese rule, is considered to have began with the first arrival of the Portuguese navigators in the land, in 1500, and to have ended with the national independence, in 1822 (see, e.g., (Klein and Luna, 2010), see also (Wikipedia, 2019)).

From the XVI century, the economy of the northeast region of Brazil was based on *slavery-based sugar plantations*, operating as sugar exporters for the metropole in Lisbon, during the colonial period, and to Europe in general, after the independence.

From the end of the XVIII century up to the legal abolition of slavery, in 1888, the economy of the southern region of Brazil was based on *slavery-based charque factories*, called *charqueadas*, which exported charque (*beef jerky*) mainly for those northeastern plantations, to serve as slave food. By the end of the XIX century, that economy reached its exhaustion, and failed to continue profitable even before the legal abolition of slavery.

In the next subsection, we formally analyze, if in a very sketchy way, the fall of the slavery-based economic system of the southern region of colonial Brazil.

7.2. A Formal Account of the Fall of the Slavery-Based Economic System of Southern Colonial Brazil

This section builds on elements of the classical socio-political-economic study of the slavery-based economy of southern colonial Brazil, by F. H. Cardoso (Cardoso, 1997). Figure 2 shows the main economic and political actors operating in that system. The *northeastern Brazilian plantations* were the consumers of the charque produced by the *southern Brazilian charque producers*. We denote their economic exchanges (arrow A in Fig. 2) simply by:

$$BCharPlantExch^t = \langle BChar\,/\,charque \rangle^t \rightleftarrows \langle Plant\,/\,money \rangle^t$$

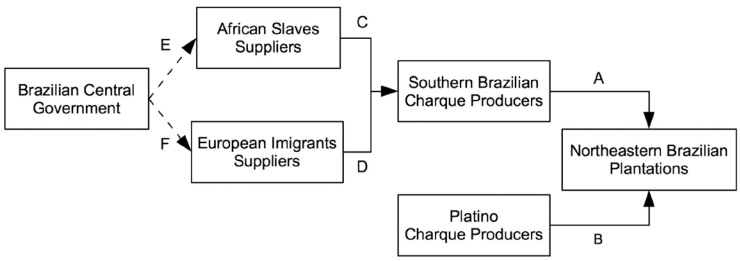

Figure 2: Sketch of the situation of the southern Brazilian charque producers.

The *Platino charque producers* were charque producers located in the neighbor countries of Uruguay and Argentina, on the shores of the Plata river. In opposition to the Brazilian producers, they operated with free labor-based industries, and consequently were strong competitors of the Brazilian producers. We denote their economic exchanges (B) with the northeastern Brazilian plantations by:

$$PCharPlantExch^t = \langle PChar\,/\,charque \rangle^t \rightleftarrows \langle Plant\,/\,money \rangle^t$$

African slave suppliers arrived in Brazil as early as the XVI century. They flourished until the middle of the XIX century, when slave traffic was legally forbidden. Their exchanges with the charque producers is denoted by:

$$SSuppPCharExch^t = \langle SSupp\,/\,slaves \rangle^t \rightleftarrows \langle PChar\,/\,money \rangle^t$$

The continuous supply of slaves for the charque producers was a crucial issue in their business, since a continuous renovation of slaves was required, given that slaves were useful as workers for relatively short time, due to the hard conditions of their jobs. Additionally, the increase of the number of slaves was the only way to increase the amount of charque production, when the demand for that product increased, due to the low level of technology of that industry.

For most of the time the charque industry was in operation, the the adoption of a slave-based organization of the production was a rational choice, that is, for the typical Brazilian charque producer it used to hold that:

$$\sum_{j=1}^{j=n} costenforc(BChar, slave_j) \leq \sum_{k=1}^{k=m} costenforc(BChar, freelaborer_k)$$

That changed, however, when the legal supply of slaves finished and the black market of slave supplies soon became risky and overpriced. The adoption of a free labor-based organization of their charqueadas immediately presented itself. But, since the slavery system was so generalized in Brazil, no stimulus had occurred in its history for the formation of free laborers for any of the main economic sectors of the county.

The alternative for most of those sectors (specially for the coffee plantations) was the importation of foreign workers, mainly from Europe, which was increasingly being stimulated by the Brazilian central government.

That was no applicable to the charqueadas, however, due to too main factors: the low technological level of that industry imposed that work could be done there only under a severe discipline that was incompatible with a free labor-based organization. Also, the charque producers themselves were culturally attached to the idea of slave labor, that they could not accept the social and legal rules demanded by free workers. In addition, the Platino charque producers had improved their competition power, imposing an increase in the productivity of the Brazlian charque producers.

The neat result of the situation was that the Brazilian charque producers became trapped in a blind alley, because it still hold for them that the cost of enforcement of slave based-production was less than the cost of enforcement of free labor-based production, but: (1) the shortage of the slaves supplies was a definitive condition, and (2) the difficulties they faced to hire free foreign workers.

The fall of the slave based-economic system of southern Brazil, centered around the charqueadas, was inevitable, and by the end of the XIX century that industry had virtually disappeared. The legal abolition of slavery, in 1888, came just to put a formal end to it.

8. Conclusion

This paper introduced elements for the agent-based modeling of *slavery-based economic system*. Making use of the formal model of *material agent societies*, a formal account was given of basic aspects of slavery-based economic systems: chattel slavery, master-slave property relation, master-slave economic exchange, slavery- based economic exchanges, slavery-based economic processes, slavery-based economic system. Basic aspects of slavery-supporting legal systems were also formally presented.

As a first case study, North & Thomas' model for the rational choice between *slavery-based* and *free labor- based* elementary economic systems was shown to be applicable to material agent societies. As a second case study, elements of F. H. Cardoso's study of slavery in the south of Brazil were used to formally characterize aspects of the slave based-charque industry.

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