



Decentralized Coalition Formation with Agent-based Combinatorial Heuristics

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KEYWORD

Smart Grid;
Coalition
Structure
Generation;
Surplus
Distribution;
Decentralized
Heuristics

ABSTRACT

A steadily growing pervasion of the energy distribution grid with communication technology is widely seen as an enabler for new computational coordination techniques for renewable, distributed generation as well as for bundling with controllable consumers. Smart markets will foster a decentralized grid management. One important task as prerequisite to decentralized management is the ability to group together in order to jointly gain enough suitable flexibility and capacity to assume responsibility for a specific control task in the grid. In self-organized smart grid scenarios, grouping or coalition formation has to be achieved in a decentralized and situation aware way based on individual capabilities. We present a fully decentralized coalition formation approach based on an established agent-based heuristics for predictive scheduling with the additional advantage of keeping all information about local decision base and local operational constraints private. Two closely interlocked optimization processes orchestrate an overall procedure that adapts a coalition structure to best suit a given set of energy products. The approach is evaluated in several simulation scenarios with different type of established models for integrating distributed energy resources and is also extended to the induced use case of surplus distribution using basically the same algorithm.

1. Introduction

Across Europe, especially in Germany where currently a financial security of guaranteed feed-in prices is given, the share of distributed energy resources (DER) is rapidly growing. Following the goal defined by the European Commission (European Parliament & Council, 2009), a concept for integration into electricity markets is needed for both: active power provision and ancillary services (Abarategui *et al.*, 2009; Nieße *et al.*, 2012) to reduce subsidy dependence. A well-known concept for aggregating DER to a jointly controllable entity is known as virtual power plant (VPP). Apart from controlling distributed electricity generation, e.g. combined heat and power (CHP), photovoltaic or wind power, controllable consumption like shiftable loads, heat pumps or air conditioning might also be included for planning active power schedules. Battery storages are a much-discussed subject to complement such groups of DER.

Virtual power plants are a well-known means for aggregating and controlling DER (Awerbuch and Preston, 1997) and their requisite integration into current and future market structures recently led to VPP systems that frequently re-configure themselves for a market and product-led alignment (Nieße *et al.*, 2014). Based on the size



of products from the order book (e. g. traded at a day-ahead market), coalitions of usually small energy resources have to be found that may apply for the product, as single units often do not have a sufficient power level nor the flexibility to fulfill the product alone. The same will hold true for other energy products; for instance for control power. In scenarios with self-dependently operated units that trade their power independently, consequentially self-organizing algorithms are required also for coalition formation to find potential partners specifically for the current product at hand.

In general, distributed control schemes based on multi-agent systems are considered advantageous for large-scale problems as expected in future smart grids due to the large number of distributed energy resources that take over control tasks from large-scale central power plants (Nieße *et al.*, 2012). Some recent implementations are (Hinrichs *et al.*, 2013a; Ramchurn *et al.*, 2011; Kamphuis *et al.*, 2007). Distributed organization and self-organized control is also a special characteristic of dynamic virtual power plants (Nieße *et al.*, 2014) whereof efficient coalition formation is an essential requirement, but with hard combinatorial specifications.

We adapted an approach for decentralized predictive scheduling (COHDA) (Hinrichs *et al.*, 2014) that already rendered efficient for decentralized combinatorial problems and that provides the additional advantage of keeping all information about local decision base's and local operational constraints private. In (Bremer and Lehnhoff, 2016) it has been further developed to decentralized coalition formation and here we extend the idea to fair surplus distribution.

The rest of the paper is organized as follows. As a start, a review of related approaches with focus on applications to the smart grid is given. We then introduce the concept of basic COHDA and our extensions to the use case of coalition formation. Finally, a conclusion is drawn with results from several simulation runs that prove applicability. In addition, we extend the ideas of COHDA and the adaption to coalition structure generation to fair decentralized surplus distribution within coalitions after product delivery.

2. Related Work

Market integration of virtual power plants sometimes demands for dynamic aggregation when it comes to energy product centric clustering of units for better trading positions (Nieße *et al.*, 2014).

In order to address the integration of the current market situation, (Nieße *et al.*, 2014) introduces the concept of a dynamic virtual power plant (DVPP). In this approach VPPs gather dynamically together with respect to concrete electricity products at an energy market. Coalition will will diverge right after delivery and reform for the successive planning period. Grouping sets of agents together so as to form coalitions is one of the major interaction concepts within the field of multi-agent systems. The usual reason for grouping agents is that a group of agents jointly might achieve a goal better than a single agent (Mas-Colell *et al.*, 1995; Kahan and Rapoport, 1984). Often, having a group of agents is indispensable in the smart grid field because, a single energy unit usually has not the capability to achieve a certain goal alone. Nevertheless, it is possible to have different alternative groups consisting of a different mixture of agents. In more complex scenarios where different groups are needed at the same time for different control tasks or product delivery, different structures of groups of groups are also possible.

Clearly, each agent (or the associated real unit) possesses certain traits or benefits that contribute to the overall success of the coalition with different amount. In some use cases, the utility of the members of a coalition may be easily quantified. For example, (Hsu and Soo, 2009) studies several examples from the transport sector; with numbers of drivers or trucks and costs expressed in dollar. In the use case studied in this paper, the utility has to be expressed as the contribution that eases a joint planning problem (jointly planning individual loads for gaining a desired aggregated schedule) that has to be expressed in terms of traits and size of individual search spaces of alternative schedules. Thus, each agent contributes an abstract flexibility to the joint task. The actually chosen schedule is not necessarily proportional to the agent's utility, because the richness of offered opportunities (moreover in case of having backup capabilities for later re-scheduling) has to be considered, not the taken choice. The role of integrating whole flexibility considerations into decentralized value calculations has for example been studied in (Bremer and Sonnenschein, 2013b).

For a self-organized partitioning in such volatile environment, a controlling agent needs a means to generate a set of feasible schedules to choose appropriate ones for negotiation at market. In order to implement agents that are able to control and represent arbitrary energy units, a technique is needed that is able to generate feasible schedules without knowledge on the actual operation of the device, cost functions or technical constraints of the controlled unit.

Classical approaches to coalition formation (Rahwan *et al.*, 2015; Gensollen *et al.*, 2015; Vinyals *et al.*, 2012; Bistaffa *et al.*, 2012) are either not decentralized and thus do not fully comply with requirements resulting from the large number of decentralized energy resources (Nieße *et al.*, 2012), do not take into account individually flexibilities and operation constraints of individual units or rely on integrated models and thus on specific, restricted implementations. Other approaches specialize to sub use cases, e. g. electric vehicles (Bazzan and de O. Ramos, 2015), to ease computation.

As opposed to the here scrutinized distribution of joint effort for value maximization, distributing joint cost is a long discussed topic (Friedman and Moulin, 1999; Bremer and Sonnenschein, 2013b). The division of surplus has applications in production (Friedman and Moulin, 1999; Sen, 1966), electricity pricing (Lima *et al.*, 1995), public goods (Mas-Colell, 1980; Champsaur, 1975), and other situations modeled by cooperative games. A survey of coalition games applied to several use cases in the smart grid can be found in (Saad *et al.*, 2012); e.g. some utility functions (analytically calculable) are given, but profit distribution is not discussed.

In recent years, researchers have investigated a variety of approaches to coalition structure generation as a major challenge in multi-agent systems (Rahwan *et al.*, 2014). An overview can for example be found in (Rahwan *et al.*, 2015). The main problems in all solutions arise with execution performance, solution quality or memory requirements (Rahwan *et al.*, 2014). Several algorithms have also been developed for coalition formation especially in the smart grid; e. g. (Gensollen *et al.*, 2015; Vinyals *et al.*, 2012; Bistaffa *et al.*, 2012). The focus in this works lies on constructing stable, i. e. long term, coalitions whereas dynamic reconfigurations are hardly supported. Individual DER models for determining volatile, individual sets of situation-dependent flexibilities are not integrated. All approaches from the smart grid field are either not decentralized and thus will hardly cope with the expected large number of decentralized energy resources (Nieße *et al.*, 2012), do not take into account the set of individual, unit specific sets of alternative capabilities or rely on integrated models and thus on specific, restricted implementations.

A first formal model for a fully self-organized solution has been presented in (Beer and Appelrath, 2013). In general, decentralized algorithms are considered advantageous in many fields of smart grid computation (Nieße *et al.*, 2012; Ramchurn *et al.*, 2012). For the case of predictive scheduling, (Hinrichs *et al.*, 2014) developed a decentralized algorithm for constrained combinatorial problems. Combined with an appropriate abstraction from individual flexibilities (Bremer and Sonnenschein, 2013a), it has been successfully applied to dynamic day-ahead market scenarios (Nieße *et al.*, 2014) for intra coalition optimization. In (Bremer and Lehnhoff, 2016) this approach has been extended to the use case of decentralized coalition formation.

3. Algorithm

3.1. COHDA

We start with a brief recap of the used base algorithm. The Combinatorial Optimization Heuristics for Distributed Agents (COHDA) was originally introduced in (Hinrichs *et al.*, 2013b; Hinrichs *et al.*, 2014). Since then it has been applied to a variety of smart grid applications (Hinrichs *et al.*, 2013a; Nieße *et al.*, 2014; Nieße and Sonnenschein, 2015; Sonnenschein *et al.*, 2015b). With our explanations we follow (Hinrichs *et al.*, 2014).

Originally, COHDA has been designed as a fully distributed solution to the predictive scheduling problem (as distributed constraint optimization formulation) in smart grid management (Hinrichs *et al.*, 2013b). In this scenario, each agent in the multi-agent system is in charge of controlling exactly one distributed energy resource (generator or controllable consumer) with procurement for negotiating the energy. All energy resources are drawn together to a virtual power plant and the controlling agents form a coalition that has to control the

VPP in a distributed way. It is the goal for the predictive scheduling problem to find exactly one schedule for each energy unit such that

1. each assigned schedule can be operated by the respective energy unit without violating any hard technical constraint, and
2. the difference between the sum of all targets and a desired given target schedule is minimized.

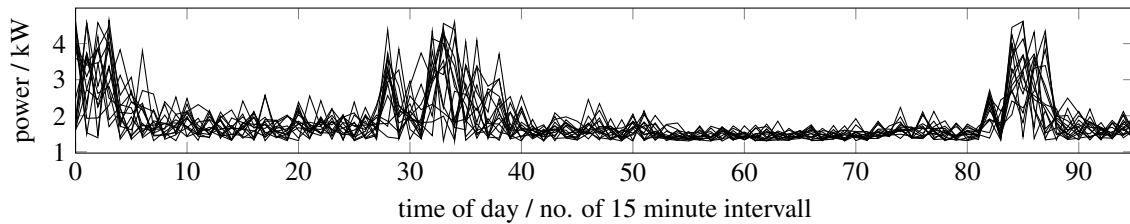


Figure 1. Example for a discrete version of the search space for a co-generation plant. Usually, 2000 different, operable schedules to choose from are used in this contribution. A state-of-charge of 50% at night and an increased thermal demand for showering in the morning and dish washing in the evening result in higher flexibilities during these periods

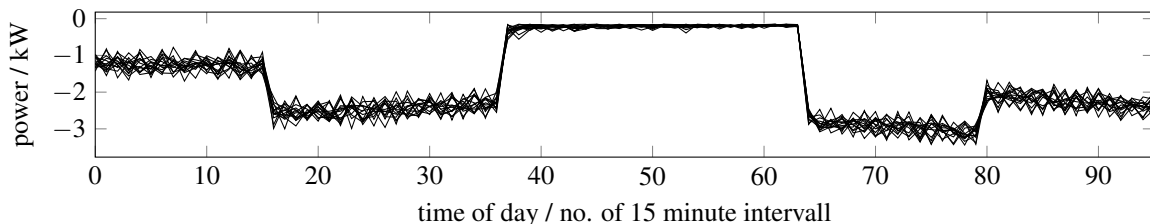


Figure 2. Example for a training set of schedules for a heat pump with a maximum deviation of 500Wh from the integral of set thermal demand

The target schedule usually comprises 96 time intervals of 15 minutes each with a given amount of energy (or equivalently mean active power) for each time interval, but might also be constituted for a shorter time frame by a given energy product that the coalition has to deliver.

An agent in COHDA does not represent a complete solution as it is the case for instance in population-based approaches (Poli *et al.*, 2007; Karaboga and Basturk, 2007). Each agent represents a class within a multiple choice knapsack combinatorial problem (Lust and Teghem, 2010). Applied to predictive scheduling each class refers to the feasible region in the solution space of the respective energy unit. Each agent chooses schedules as solution candidate only from the set of feasible schedules that belongs to the DER controlled by this agent. Each agent is connected with a rather small subset of other agents from the multi-agent system and may only communicate with agents from this limited neighborhood. The neighborhood (communication network) is defined by a small world graph (Watts and Strogatz, 1998). As long as this graph is at least simply connected, each agent collects information from the direct neighborhood and as each received message also contains (not necessarily up-to-date) information from the transitive neighborhood, each agent may accumulate information about the choices of other agents and thus gains his own local belief of the aggregated schedule that the other agents are going to operate. With this belief each agent may choose a schedule for the own controlled energy unit in a way that the coalition is put forward best while at the same time own constraints are obeyed and own interests are pursued.

All choices for own schedules are rooted in incomplete knowledge and beliefs in what other agents do; gathered from received messages. The taken choice (together with the basis for decision-making) is communicated to all neighbors and in this way knowledge is successively spread throughout the coalition without any central

memory. This process is repeated. Because all spread information about schedule choices is labeled with an age, each agent may decide easily whether the own knowledge repository has to be updated upon message reception. Any update results in recalculating of the own best schedule contribution and spreading it to the direct neighbors. By and by all agents accumulate complete information and as soon as no agent is capable of offering a schedule that results in a better solution, the algorithm converges and terminates. Convergence has been proved in (Hinrichs, 2014).

More formally, each time an agent receives a message, three successive steps are conducted. First, during the perceive phase an agent a_j updates its own working memory κ_j with the received working memory κ_i from agent a_i . From the foreign working memory the objective of the optimization (i. e. the target schedule) is imported (if not already known) as well as the configuration that constitutes the calculation base of neighboring agent a_i . An update is conducted if the received configuration is larger or has achieved a better objective value. In this way, schedules that reflect the so far best choices of other agents and that are not already known in the own working memory are imported from the received memory.

During the following decision phase agent a_j has to decide on the best choice for his own schedule based on the updated belief about the system state γ_k . Index k indicates the age of the system state information. The agent knows which schedules of a subset (or all) of the schedule that other agents are going to operate. Thus, the schedule that fills the gap to the desired target schedule exactly can be easily identified. Due to operational constraints of the controlled DER, this optimal schedule can usually not be operated. In addition, other reasons might render some schedules largely unattractive due to high cost. Because of this reason, each agent is equipped with a so called decoder that automatically maps the identified optimal schedule to a nearby feasible schedule that is operable by the DER and thus feasible. Based on a set of feasible schedules sampled from an appropriate simulation model for flexibility prediction (Bremer and Sonnenschein, 2013c), a decoder can e. g. be based directly on this set (by linearly searching the schedule with the smallest deviation) or be built by learning a support vector model after the approach of (Bremer and Sonnenschein, 2013a). Both approaches – with individual advantages and drawbacks regarding computational complexity, search space size and accuracy – have been tested here.

Figure 1 shows an example training set of 96-dimensional schedules taken from the model of a co-generation plant with simulated detached house defining a heat demand based on weather induced losses and hot water drawing profiles. Figure 2 shows a second example with a training set derived from a heat pump model. Such training sets define the individual flexibility of an energy resource for a given time horizon (in this case 96 intervals of 15 minutes each) and serve as the stencil from which the abstract machine learning model as well as the simple linear model and the decoder for generating automatically feasible schedules are derived.

As the whole procedure is based exclusively on local decisions, each agent decides privately which schedules are taken. Private interest and preferences can be included and all information on the flexibility of the local DER is kept private.

If the objective value for the configuration with this new candidate is better, this new schedule is kept as selected one. Finally, if a new solution candidate has been found, the working memory with this new configuration is sent to all agents in the local neighborhood upon which these recipients conduct the same process for themselves. The procedure terminates, as soon as all agents reach the same system state, no better solutions are found, and no new messages are generated. Then all agents know the same final result.

3.2. Decentralized Coalition Formation

In general, a set of units has to be assigned to a given electricity product such that the assigned group is capable of jointly operating the target schedule defined by the product. If a set of products is given, the set of all available energy units has to be divided out to the different products in a way that the outcome (generated by product delivery) for the units and product fulfilment are maximized concurrently. The problem of partitioning a set of agents into several groups is a coalition structure generation problem (Nieße *et al.*, 2014). Given is a set of products \mathcal{P} . Each product $P \in \mathcal{P}$ defines a schedule $P = (p_1, \dots, p_d) \in \mathbb{R}^d$ that defines for each time interval i a value p_i of mean active power that is to be delivered as close as possible. Usually, 15 minute time intervals for a given product horizon d (up to 96 intervals) are considered. We do not consider the origin of the products.

Energy products might be drawn from some energy market like active power or reserve control markets. In this contribution we restrict our simulations w.l.o.g. to day-ahead active power trading but take all products as already given from some earlier market phase. In addition to the product set \mathcal{P} a set of agents \mathcal{A} is given.

We want to find a coalition structure $CS = (\mathcal{A}_1, \dots, \mathcal{A}_{|\mathcal{P}|})$ such that the following implication holds: $A_i \in \mathcal{A}_j \Rightarrow A_i \notin \cup_{k \neq j} \mathcal{A}_k$, i.e. each agent may only be part of exactly one coalition but does not necessarily have to join a coalition. Usually in coalition structure formation one wants to maximize the profit of the coalition structure. In our use case we want to equivalently minimize the deviation from the coalition's ability to fulfil their assigned energy product and thus define the value $v(CS)$ of a coalition structure CS as

$$v(CS) = \sum_{j=1}^{|\mathcal{P}|} \delta \left(\sum_{A \in \mathcal{A}_j} x_A, P_j \right) \rightarrow \min \quad (1)$$

such that

$$x_{A_i} \in \mathcal{F}_{U_i}, \forall A_i \in CS. \quad (2)$$

Eq. (1) defines the value $v(CS)$ as the sum of minimal deviations δ (w.l.o.g. the Euclidean distance has been used) between a coalition's best schedule choice and the respective target product. This means, each coalition gathers flexibilities from different DER such that each coalition can distribute the effort of jointly achieving the product to feasible schedules for the respective coalition members. As a constraint (2), schedules must be taken from the subset \mathcal{F}_{U_i} of feasible schedules. Each unit U_i defines its individual feasible region based on individual technical and economical restrictions and on current operational state and forecasts (e. g. weather, etc.).

COHDA can easily conduct different types of combinatorial optimization if the decision phase is appropriately adapted to the problem at hand. Some minor changes for objective evaluation and working memory content are additionally necessary. We adopted COHDA for solving the coalition formation problem as follows. The system configuration γ_i denotes the assignment of each (so far known) agent to a coalition; i. e. each agent may assign itself to exactly one coalition (denoted by the index of the chosen product) or may choose not to be a member of any coalition (index 0). After receiving an updated system configuration with assignments from other agents, a decision has to be made on the best own assignment. To achieve this, the agent tests all possible assignments for its own DER for finding the one with the smallest error:

$$\arg \min_{0 \leq i \leq |\mathcal{P}|} v(CS_{a_j \in \mathcal{A}_i}). \quad (3)$$

$CS_{a_j \in \mathcal{A}_i}$ denotes a coalition structure with agent a_j being a member of coalition i (or not in a coalition if $i = 0$). The value of the coalition is given by the residual error Eq. (1). Thus, the better the ability of the coalitions to resemble the target schedules defined by the associated products, the better the coalition structure. In this way, evaluating the objective (3) comprises a number of inner optimization processes. Evaluating an assignment necessitates solving the optimization problem of finding best schedules for each coalition in order to calculate Eq.

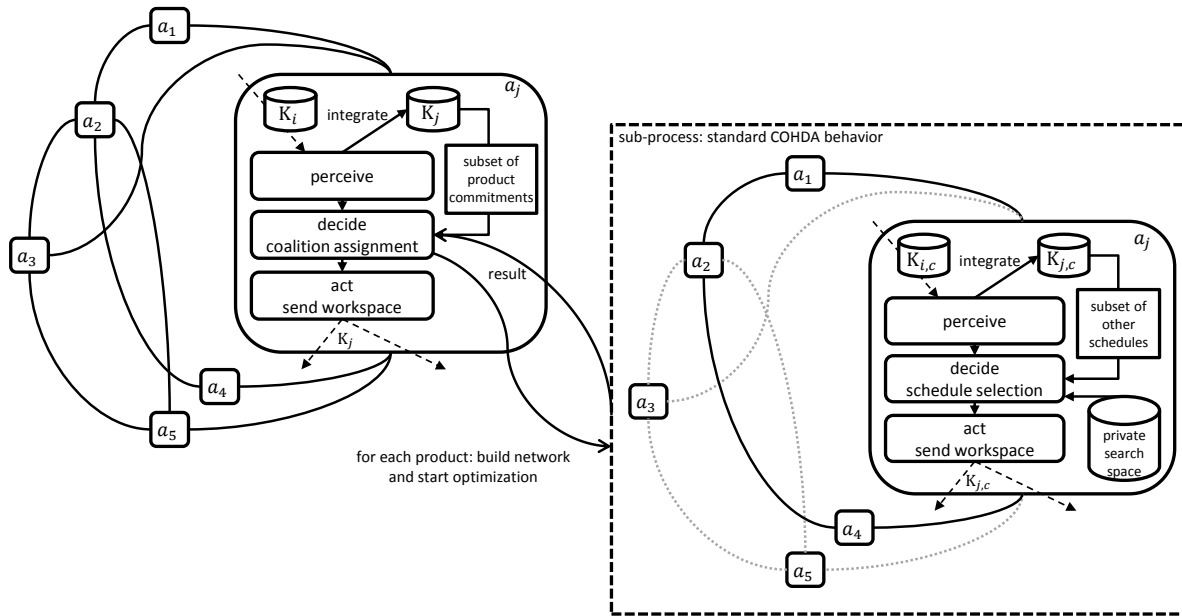


Figure 3. Basic architecture and process of coalition formation as nested decentralized combinatorial optimization

(1). This task is achieved by starting standard COHDA processes as described in (Hinrichs *et al.*, 2014) on the respective subset of agents (belonging to the coalition at question) and a respectively smaller communication overlay network. Figure 3 illustrates the overall process and the interaction between the different COHDA instances. These COHDA behaviours are interlaced with the main coalition formation COHDA behaviour. Thus, working memories $\kappa_{j,c}$ are distinguished by an additional identifying key c ; cf. Figure 3.

Two interlaced and interacting COHDA processes do the work as shown in Figure 3. One COHDA process (the outer) determines during its decision phase the best assignment to a coalition by probing each possible one (left part of Figure 3). Evaluating an assignment of one's own controlled energy resource to a coalition (a belief about the assignment of other agents is derived combining the information from received messages) incorporates a need for solving a predictive scheduling optimization problem for each assignment. In order to solve this (inner) optimization problem, the outer process is paused and a new COHDA process is initiated by emitting a message to an agent from the neighborhood. The inner process comprises only agents from the currently evaluated coalition and thus the neighborhood is smaller (right side of Figure 3 that illustrates the internal process of the decision stage of the outer process). When the inner optimization process terminates with a value of the tested coalition, the next inner evaluation process is started. After all possible assignments have been tested, the best is chosen and the outer process continues.

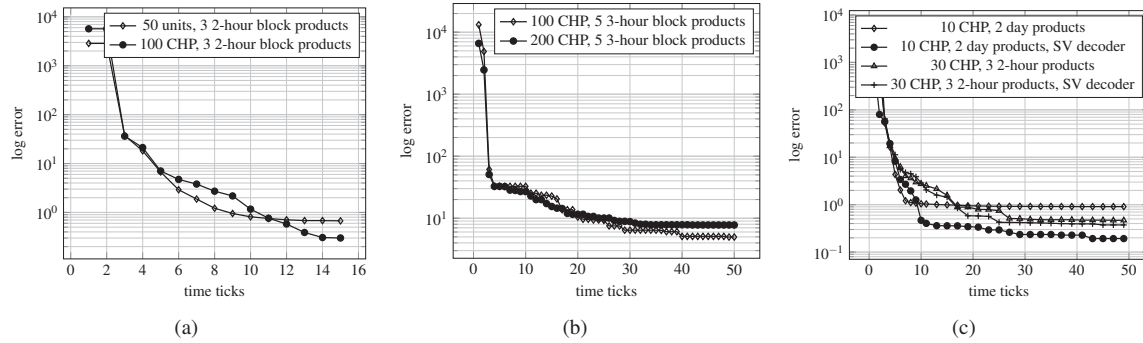


Figure 4: Mean convergence behavior of several test runs (100 each) for different scenarios for the first execution steps of the outer combinatorics. The intermediate best solution quality was collected during executing the multi-agent system with the help of a global observer. This data collection is not part of the otherwise fully decentralized algorithm

4. Results

The evaluation of our approach was done by simulation experiments. In our test setting, each energy unit is associated with a controlling agent responsible for conducting local decisions and communication with other agents. For simulating distributed energy resources we used a model for co-generation plants that has already served in several studies and projects for evaluation (Bremer *et al.*, 2010; Bremer and Sonnenschein, 2013a; Neugebauer *et al.*, 2015; Hinrichs *et al.*, 2013a; Nieße and Sonnenschein, 2015). This model comprises a micro CHP with 4.7 kW of rated electrical power (12.6 kW thermal power) bundled with a thermal buffer store. Constraints restrict power band, buffer charging, gradients, minimum runtime, and satisfaction of thermal demand. The relationship between electrical (active) power and thermal power was modeled after engine test benches. Thermal demand is determined by simulating a detached house (including hot water drawing) according to given weather profiles. In addition, we used models for heat pumps and boilers for hot water provision (Sonnenschein *et al.*, 2015a). For each agent the model is individually (randomly) configured with state of charge, weather condition, temperature range, allowed operation gradients, and similar. From these model instances, the respective training sets for building the decoders have been generated with the sampling approach from (Bremer and Sonnenschein, 2013c).

Figure 4 shows the results of some first convergence experiments with coalitions of co-generation plants. In order to ensure the reproducibility of results as well as the measurability of convergence, an event-based simulation of our multi-agent system with a discrete time model and with asynchronous execution within each time step has been used for testing. Shown are the achieved mean residual errors. Please note that in each of these time steps of the outer algorithm a different number of agents has to conduct the inner (predictive scheduling algorithm) several times in order to evaluate the value of each possible coalition structure during the

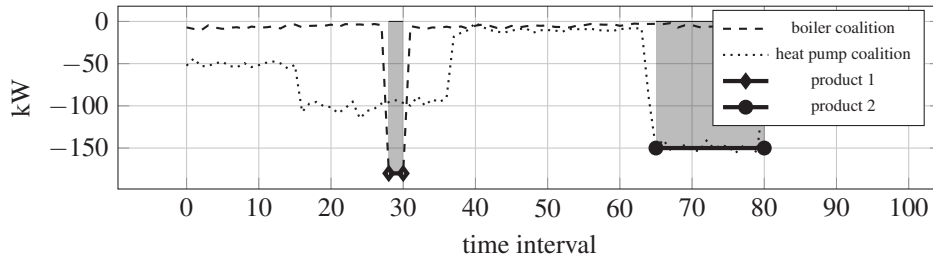


Figure 5. Example result for a scenario with two block products. One (30 minutes in the morning) tailor made for the boilers in the agent set and one in the evening suitable for the heat pumps. Without knowing the unit under control the set of agents splits apart correctly and schedules are assigned (taking also into account the operation of the rest of the day) such that product fulfilment is maximized while all technical constraints are obeyed

decision phase. All simulations have been stopped early after 15 and accordingly 50 steps to demonstrate the fast convergence. The omitted improvements during later time steps are rather marginal.

Figure 4(a) shows the results of two easy problem instances with 50 and 100 units and 3 block products for a 2 hour period (resolution: 15 minutes) as for example traded at the European Power Exchange¹. The height of the load has been chosen with respect to the mean operated power of all CHP. Nevertheless, it is not guaranteed that a coalition can be found for each product such that it is possible to find a schedule for each unit in the coalition such that the aggregated schedule of all units in the coalition resembles the product exactly. As error measure, the mean absolute percentage (MAPE) measure

$$d_M = \frac{100}{d} \sum_{i=1}^d \left| \frac{p_i - x_i}{p_i} \right| \quad (4)$$

has been used for products over d time periods with product power p_i and joint coalition power x_i at interval i . Scaled in this way, results with different number of units and power levels can be compared by the mean deviation in percent from the wanted power. MAPE allows comparing coalitions of different size. Please note that minimizing some error for the coalition structure is equivalent to maximizing some given revenue measure. Figure 4(b) shows results for more complex scenarios. Figure 4(c) takes into consideration products for a whole day and uses the support vector decoder as search space model instead of the discrete model. Here an exact result was achievable if the right CHPs are in the right coalition and the schedules are chosen appropriately.

Figure 5 shows an example result for a scenario with two different block products: one tailor made for the heat pumps in the scenario and one suitable for the boilers (although again an exact solution was not achievable). The set of all agents is split up into two coalitions (not being in a coalition was again an allowed option for an agent) while flexibilities of all units for the whole day had to be kept in mind when calculating possible contributions to the shorter intra-day products. The figure shows the aggregated schedules for the whole day and the two products.

1. <http://www.epexspot.com>.

Table 1. Results of different coalition formation scenarios. The residual error is given as MAPE d_M Eq. (4). Evaluations are counted as number of decisions on coalition assignment (outer) and decision on optimal schedule choice (inner)

| scenario | error | outer evaluations | inner evaluations |
|----------|---|-----------------------|----------------------------|
| CHP | 0.24971 ± 0.23290 | 2655.31 ± 1450.40 | 176539.42 ± 101171.13 |
| HP | $1.1545 \times 10^{-3} \pm 2.2155 \times 10^{-4}$ | 3864.07 ± 1492.07 | 299072.67 ± 115364.16 |
| Mixed 1 | $4.5084 \times 10^{-15} \pm 8.3635 \times 10^{-16}$ | 6494.50 ± 3776.21 | 663890.85 ± 417128.31 |
| Mixed 2 | 0.08572 ± 0.34291 | 9908.12 ± 3157.91 | 1115235.44 ± 379695.34 |

In Table 1 several scenarios are compared with regard to the achieved result quality and the used number of objective evaluations. The scenarios are:

CHP: A scenario with 10 agent controlled CHP with randomly chosen initial operation state regarding state of charge, weather condition and possible gradients. Two coalitions for two products covering a time range of 96 intervals of 15 minutes each are sought. Both products have been chosen such that an exact fulfilment (zero error) could be achieved. Support vector decoders have been used as search space models.

HP: This is basically the same scenario as for the CHPs but with 10 heat pumps. This time, the discrete model has been used as search space.

Mixed 1: This scenario consists of 5 CHP, 5 heat pumps, 5 boilers and 3 products. Each product was tailor made for one of the groups of units. Thus, an error of zero was achievable. For modeling the search space, the discrete model with 100 different possible schedules for each unit has been used.

Mixed 2: The setup for this scenario was the same as for Mixed 1. This time a discrete model with 200 schedules had been used for each unit; resulting in an increase in problem size.

Table 1 shows the residual error (MAPE d_M) that has been achieved together with the number of conducted evaluations of the objective function distinguished in outer (agent decides on coalition assignment) and inner (agent decides on appropriateness of the schedule selection within a questioned coalition) objective.

5. Surplus Distribution

In single owned VPP it is quite easy to establish a centralized control instance and moreover: no need arises for a distribution of any generated surplus. In case of a coalition of temporarily pooled but individually owned and operated units (as expected in smart grid scenarios), on the other hand, the question for a fair distribution of jointly earned profit has not yet sufficiently been answered. Self-organized coalitions on all accounts need a scheme for self-organized and thus decentralized surplus distribution.

Each agent possesses certain traits or benefits that contribute differently to the overall success of the coalition. In some use cases, the utility of the members of a coalition may be easily quantified. For example, (Hsu and Soo, 2009) studies several examples from the transport sector with numbers of drivers or trucks and costs expressed in dollar. In the use case studied in (Bremer and Sonnenschein, 2013b), the utility has to be expressed as the contribution that eases a joint planning problem (jointly planning individual load for gaining a wanted aggregated schedule) what has to be expressed in terms of traits and size of individual search spaces of alternative schedules and thus in terms of individual flexibility. The actually chosen schedule is not necessarily proportional to the utility of the unit because the richness of offered opportunities (moreover in case of having backup capabilities for later re-scheduling) has to be considered; not the taken choice.

Distributing joint cost is a long discussed topic (Friedman and Moulin, 1999). The division of surplus has use cases and applications in production (Friedman and Moulin, 1999; Sen, 1966), electricity pricing (Lima *et*

al., 1995), public goods (Mas-Colell, 1980; Champsaur, 1975), and other situations modeled by cooperative games. Cooperative game theory is concerned with problems from coalition formation in multi agent systems and desirable properties of such coalitions (Fatima *et al.*, 2008; Rapoport, 1970). Among others, cooperative game theory offers concepts for fair distribution of jointly earned gains among all members of a coalition. The Shapley value (Shapley, 1953) is such a concept for fair payoff distribution (Hsu and Soo, 2009).

If cooperative behaviour is enforced by offering better payoff for an agent within coalitions (groups of players), the competition is between coalitions rather than between individual agents. Such game is said to be a coalition game. If the utility may be exchanged between players (for example by side payments) the game is said to have a transferable utility.

Let N be a set of n players (agents). A coalition game (N, v) with transferable utility for the player set N is characterised by a function v that measures the worth $v(S)$ of any non-empty subset $S \subseteq N$. In this sense, $v(S)$ denotes the total jointly earned payoff or benefit of a coalition S , that can be distributed among all members of S . Of course, each member makes an individual contribution that has to be considered when determining individual payoff.

The characteristic function v allows to assess the worth of a coalition in the context of a given scenario but gives no hints on how to share it. Shapley (Shapley, 1953) defined a value for games that exhibits certain fairness properties (Ma *et al.*, 2007). The Shapley value ϕ_i that is assigned to player i according to a given characteristic function v in a coalition game (N, v) that determines the gain is defined as

$$\phi_i(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(n-|S|-1)!}{n!} (v(S \cup \{i\}) - v(S)). \quad (5)$$

The term $v(S \cup \{i\}) - v(S)$ in (5) refers to the marginal contribution of player i to the value of the whole coalition (Liben-Nowell *et al.*, 2012).

The marginal contribution defines to what extent a player i contributes to the overall gain or phrased differently: how important player i is for the coalition. The value assigns a unique distribution of the total generated surplus among all members of a coalition based on the marginal contributions. Shapley values determine a fair distribution.

Especially, Shapley values are the only value distribution scheme that has all of the following four properties (cf. e.g. (Hsu and Soo, 2009)):

1. **Pareto-efficiency:** The total value of a coalition is distributed among the members: $\sum_{i \in N} \phi_i(v) = v(N)$
2. **Symmetry:** The value can be determined regardless of the name of the players. If for any two players i and j the following relation holds: $v(S \cup \{i\}) = v(S \cup \{j\})$ for every subset $S \subseteq N$ with $S \cup \{i, j\} = \emptyset$, then $\phi_i(v) = \phi_j(v)$.
3. **Additivity:** This property requires for any two games $\phi_i(N, v)$, $\phi_i(N, v^*)$ that the following relation holds: $\phi_i(N, v) + \phi_i(N, v^*) = \phi_i(N, v + v^*) \forall i \in N$.
4. **Zero player:** If the marginal value of a player to any possible coalition is zero, this player gains a value of zero: $v(S \cup i) = v(S) \forall S \Rightarrow \phi_i(v) = 0$.

Calculating Shapley values in general is intractable (Deng and Papadimitriou, 1994; Matsui and Matsui, 2001) because the calculation is often #p-complete (Valiant, 1979). The complexity depends on the value function v . There are some special cases where computational feasible calculations or at least approximations of Shapley values are known, e.g. supermodular games (Liben-Nowell *et al.*, 2012).

Weighted voting games are an example with computationally hard Shapley values (Fatima *et al.*, 2008). In such games a coalition gets a reward of 1 if the sum of individual weight values assigned to each agent is larger or equal to some given threshold and 0 otherwise. For a smart grid use case this could translate as follows: if a coalition has to jointly archive at least power q , each generator (or its controlling agent respectively) is assigned a weight of its generation power p_i . A generator coalition is rewarded if $\sum_i p_i \geq q$. An extension to this example could demand that the deviation of the joint power to the demanded power is below some given threshold.

Another use case would devalue larger deviations from the target schedule and thus continuously evaluate gains by the actually achieved similarity of demanded and achieved schedule.

Several methods have been proposed to approximate Shapley values for coalition games. Mann and Shapley started with proposing a method based on Monte Carlo simulations in (Mann and Shapley, 1960), but without suggesting how to draw necessary samples of coalitions; a more comprehensive analysis of sampling approaches can be found in (Bachrach *et al.*, 2008). In (Owen, 1972) a multi-linear extension approach with linear time complexity has been proposed, which has but to be weakened in practice due to approximation error (Leech, 2003); (Zlotkin and Rosenschein, 1994) proposed a method based on choosing random permutations from coalitions. In the following, a randomized approach from (Fatima *et al.*, 2008) with linear complexity is discussed.

The individual contribution to the quality of the solution that an agent offers to the coalition is the individual search space and its abilities, not the finally picked schedule. This search space determines the set (size, variability, suitability for the problem at hand, backup capacity for robustness, and diversity) of alternative choices from that an planning algorithm might choose. In case of a many-objective version of the above sketched problem, cost indicators for a cost effective choice of alternatives would also be part of the search space.

As has been mentioned, Shapley values provide an ideal means for calculating fair distribution keys. Unfortunately, they are computationally hard to determine in larger VPPs. In order to make nevertheless use of these indicators as a base for sharing the surplus, e. g. (Bremer and Sonnenschein, 2013b) used a Monte Carlo based estimation scheme after (Fatima *et al.*, 2008) for the case of VPPs.

The method basically works as follows: instead of calculating all marginal contributions to all possible sub-coalitions of a given player j , draw a random sample of n coalitions with size $1, 2, \dots, n$. The average of the marginal contributions of Player j in the random sample is taken as an approximation $\hat{\phi}_j(v)$ for the Shapley value $\phi_j(v)$. Obviously, this approximation can be achieved with linear time complexity $O(n)$.

Now, the marginal contribution of a DER for a coalition within the sketched smart grid load balancing scenario is to be defined. A VPP is first and foremost aiming at resembling a given load schedule as close as possible. For this reason, it seems appropriate to choose a metric that measures the distance between achieved load and wanted load as basis, because this metric is also used as (at least one) objective during optimization. In this way, the marginal contribution is proportional to the difference between the minimal distance that a coalition can achieve with agent j and the minimal distance achieved without agent j (Bremer and Sonnenschein, 2013b):

$$c(j, S) = \frac{1}{\delta_{\min}(P, s(j) + \sum_{i \in S} s(i))} - \frac{1}{\delta_{\min}(P, \sum_{i \in S} s(i))} \quad (6)$$

with $c(j, S)$ denoting the marginal contribution of agent j to coalition S and $\delta_{\min}(\cdot, \cdot)$ denoting the minimal distance (best product resemblance) that a coalition may achieve. The reciprocal of the distance is used, because achieving a small distance is of a bigger value for the coalition. Please note, that calculating a single marginal contribution already involves two predictive scheduling optimization tasks. The schedules $s(i)$ of agents i are chosen in a way that the coalition as a whole gets off best, e.g. in a way that the sum resembles the target load P as close as possible and are therefore the result of an optimization process. More elaborated versions are discussed in (Bremer and Sonnenschein, 2013b).

In the Monte Carlo approach, the marginal contribution (i. e. Eq. 6) from a sample of coalitions is averaged to gain an estimate for the Shapley values:

$$\hat{\phi}_j(c) = \frac{1}{n} \sum_{X=0}^{n-1} c(j, S^X), \quad (7)$$

with S^X denoting a random coalition with size X . In any case, the predictive scheduling optimization problem has to be solved multiple times during the estimation process for calculating marginal contributions.

The task of the optimization process is to determine how good (close in the sense of distance between schedules) a given coalition can adapt the joint schedule to the product. Then, one can compare how good a coalition

performs with and without a prospective member and thus how much the agent in question may put forward the group.

In (Bremer and Sonnenschein, 2013b) the decoder technique has been harnessed to allow each agent calculate his own Shapley value according to the following process: After choosing a random set of coalitions for the Monte Carlo approach mentioned above, an agent may calculate his marginal contributions by solving the respective optimization problems with and without his own contribution. In this way and with the help of the above mentioned optimization approach all calculations for calculating an agent's own Shapley value can be done by the agent itself, if each agent has access to all search space models in the coalition as decoders. The agents in (Bremer and Sonnenschein, 2013b) internally used a centralized approach. Because, a decoder abstracts from any specific DER and thus does not disclose any private information or key performance indicator, no private information is revealed directly. Nevertheless, as information might be derived from a decoder indirectly, we will here extend the idea and argue how determining such surplus distribution keys can also be determined fully decentralized using the very same approach of interlocked COHDA processes.

Each agent may draw a random set of coalitions (subsets of the existing coalition the agent is currently taking part in) and initiate a COHDA process for determining the best product fulfillment that the questioned coalition may achieve with and without the agents contribution. Thus, actually two COHDA optimization processes have to be issued for each coalition in an agents list. After averaging over the marginal contributions, each agent knows its own Shapley value. If the communication messages or rather the exchanged work memories are annotated with a key identifying the issuing agent as well as with an ID that identifies the questioned coalitions (note: the ID only has a meaning to the issuing agent), all optimization processes can be started at the same time and executed asynchronously in parallel. Simply, each agent needs – in addition to basic COHDA – a means for managing several communication networks and thus neighborhoods at the same time as opposed to managing just one single secondary network for the coalition formation process. Calculations regarding the different parallel optimization processes for Shapley value determination are done fully on circulated working memories and not on local information – accept for the current operational status of the device and the resulting restrictions for the flexibility. But, these are the same for all optimization processes and thus need no additional management.

In this way, the COHDA protocol can be easily used also for determining a fair distribution scheme when it comes to accounting at the end of life of a coalition.

6. Conclusion

We presented a fully decentralized approach to coalition formation based on the combinatorial heuristics COHDA. This approach fully relies on exchange of messages and local decisions and thus also allows for the integration of private interests. No information about the own flexibilities or any underlying models of private operation constraints have to be revealed to other agents. A major contribution is the integration of dynamically changing flexibilities by means of decoders, allowing coalitions to value traits and size of individual search spaces for specific product fulfillment.

The results are promising and show an acceptably small computational footprint due to the fully decentralized, parallel calculations. With decentralized coalition structure generation, a further building block is available for fully decentralized energy management.

Self-organized coalitions of agents from different owners that plan, produce and sell active power products jointly generated by units from different owners, need a scheme for fair distribution of jointly gained payoffs afterwards. Shapley values are a well-known means of calculating fair shares of payoff for all members of a coalition. We further have proposed a scheme for decentralized coalition formation by using interlocked gossiping based agent heuristics for determining the coalition structure and discussed the applicability of basically the same interwoven scheme for calculating the distributions keys for profit afterwards.

In this way, by extending COHDA to allow more than one interlocked instances, the whole life cycle of coalitions starting from formation according to a product portfolio, predictive product fulfillment planning up to fair energy flexibility based accounting after product delivery can be tackled decentralized with a single protocol.

7. References

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